International Journal of Scientific & Engineering Research, Volume 4, Issue 5, May-2013
ISSN 2229-5518

Nonlinear dynamic of periodically cardiac pacemaker

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Abstract—In this paper, a additional modified version of the classical van der pol oscillator (abbrivated as VdPO) is proposed and introducing the forced modified VdPO oscillator also known as self-excited oscillator. The resulting self-excited oscillator is analyzing in frequency and time domain, using phase portrait, spectrum analysis and bifurcation diagrams.

Index Terms— Action potention, Forced Van der pol oscillation, Homoclinic bifurcation, Phase Portrait, Quasi-periodic.

1 Introduction

TO development an dynamical system, the nonlinear system as been important roles. Such as VdPO, which are driven by a second-order nonlinear differential equation, can be represented in mechanically as a mass spring damper system with a nonlinear damping coefficient also equivalently in electrical system as RLC circuit with a negative nonlinear resistor, having many applications such as electronics, bio-Medical or acoustics etc.

$$\frac{d^2x}{dt^2}+\alpha(x^2-1)\frac{dx}{dt}+\omega_0{}^2x=0 \qquad (1)$$

In biological point of view we use van der pol oscillator over cardiac pacemaker due to some important reason as per following:

- Use the frequency from external source, with no effected on system amplitude.
- A relationship between VdPO and the heart was proposed by FitzHugh[1]. He proposed the generation of action potentials with an extended version of the Ven der pol equation, which is the simplification version of the Hodgkin-Huxley equation [2].

In this paper we obtain accurate result like as actual pacemaker by the help of modified forced van der pol oscillator and investigation of the heartbeat in the nonlinear dynamic system. The paper layout is: In second section describing about the function of actual cardiac action potential. In section 3 modification of van der pol oscillation. In section 4 show the results and graph behaviors of our designed oscillator. Section 5 & 6 is on discussion and conclusion respectively.

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2 Action Potential

One of the major public health problems is sudden cardic death; In the Eastern world, Mortality is one of the leading cases. Sudden cardiac death is comes from the ventricular fibrillation (VF). In the field of cardiac electrophysiology, ventricular fibrillation (VF) is a major challenge to understand that in the whole organ how events at the cellular level translate into arrhythmic behavior. Under the high-frequency excitation conditions, the beat-to-beat alternation in the refraction time (RT) which represent on the ECG as T-wave alternans, may be a harbinger of VF in the heart [3], [4], [5].

In the electrical point of view, cardiac cells fire an action potential, which consists of a rapid depolarization of the transmembrane voltage followed by a much slower repolarization process before returning to the resting potential. (fig.1)

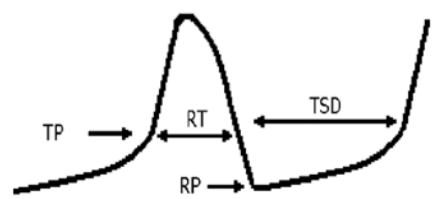

Fig. 1. Action potential of the cardiac conducting system. Where TP (threshold potential), RP (resting potential), RT (refraction time), TSD (time of spontaneous epolarization).

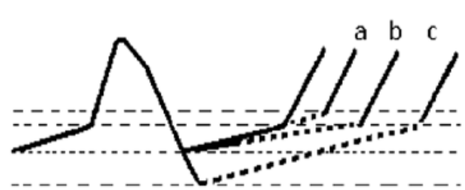

Fig. 2. Three mechanisms of changing the frequency of AC. In (a)-change of the threshold potential, (b)- change of the time of diastolic period, (c) change of the resting potential.(Taken from [9])

In generally, that mechanisms are very importent and its variation depend on many factors such as increased activity of the sympathetic or of the para-sympathetic nervous systems also can be caused by physical damage in the conducting system [6]. Inorder to make actual pacemaker the two important factors, the refractive time (RT) and the time of spontaneous depolarization (TSD), are used. According to these factors the classical van der pol oscillator does not fulfill our required condition for model of pacemaker. Fig.3 shows in time domain with different values of α. With reference to above two factors, we need to modify the classical van der pol oscillator (mVdPO).

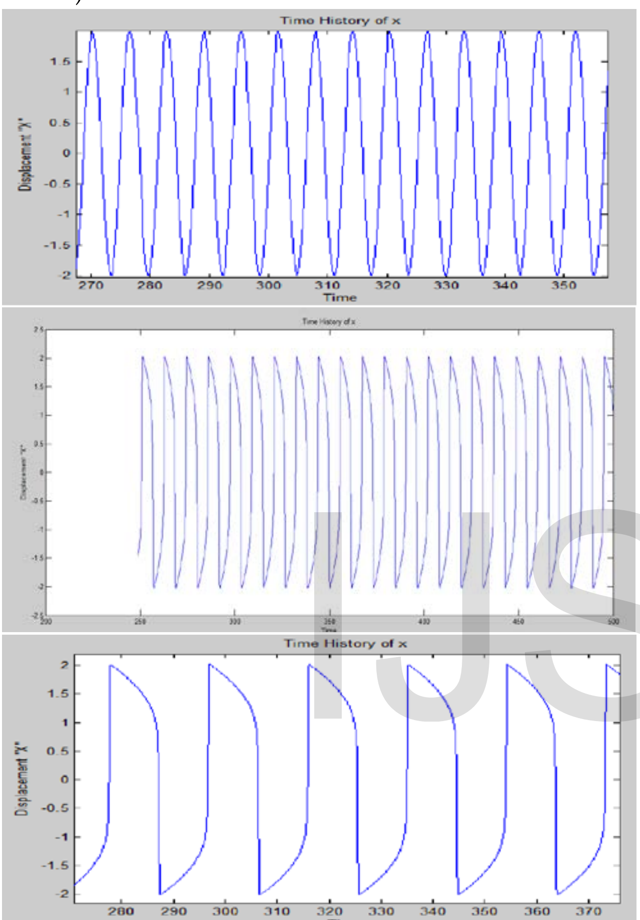

Fig. 3. Time series of VdPO Eq (1), α=1 for two values, upper 0.1, middle 5.0 and lower panel α=10.0

3 Modified van der Pol oscillator

In this oscillator the harmonic force had been replaced by during term into modified vdp oscillator [7] to focused on synchronization

$$\frac{d^2x}{dt^2}+\alpha(x^2-\mu)\frac{dx}{dt}+\frac{x(x+d)(x+2d)}{d^2}=0;d,\mu,\alpha>0;\mu<d \qquad (2)$$

The phase portrait of Eq(2) shown in Fig.4 for different values of μ, as μ increase the trajectory shape changes and move toward the negative value and appear system behavior into the node and saddle. For high value of μ, the limit cycle collide with a saddle point known as homoclinic bifurcation [8]. Such oscillation phenomena can be interpreted as a physiological effect of the actual pacemaker. Eq (2) shows that the distance between two points such as, node and saddle, cannot be changed. Introducing new parameter to modify and change distance between node and saddle [9].

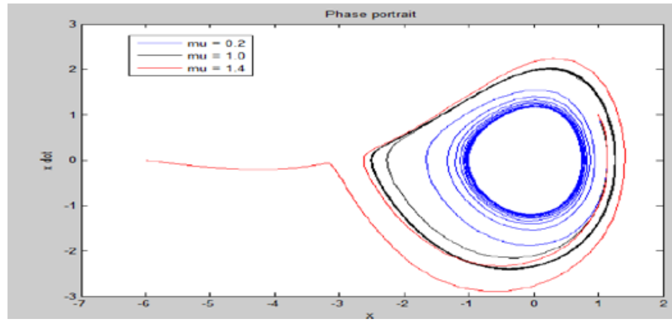

Fig. 4. Phase portrait of mVdP duffng oscillator of Eq (2) for different values of μ. The parameter α and d is 0.3 & 3 respectively.

$$\frac{d^2x}{dt^2}+\alpha(x^2-\mu)\frac{dx}{dt}+\frac{x(x+d)(x+e)}{ed}=0;d,e,\mu,\alpha>0;\mu<d \qquad (3)$$

Fig.5 for Eq (3) shows to change the depolarization period as varies the value of e. The point to be noted that in the vicinity of the saddle, the trajectory tends to send more time whenever the nodal point moves toward the saddles point. The intervals between action potential become shorter in the vice versa case [9].
In Fig.5 shows that the shape of pulses changes as changing the parameter of α in these effects the refractory time should be changed. To eliminate the change of refractory time, need to remodify of Eq (3).

$$\frac{d^2x}{dt^2}+\alpha(x-v1)(x-v2)\frac{dx}{dt}+\frac{x(x+d)(x+e)}{ed}=0;d,e,\alpha>0 \qquad (4)$$

As shown in Fig.6, by varies the value of v1 & v2, to change the resting potential thus change the frequency of the action potential.

4 FORCED VAN DER POL OSCILLATOR

The VdPO model with external excitation is driven by a non-autonomous second order differential equation as follows:

$$\frac{d^2x}{dt^2}+{\omega_0}^2-(\mu-\alpha x^2)\frac{dx}{dt}=F\cos(\Omega t) \qquad (5)$$

Where the function of ω is to control how much voltage injected into the system ans α is to control the way in which voltage flow through the system. The variable α control the rate at which the slow built up occures. As α decrease the voltage collect more slowly. Making the slow phase take longer and hence slowing down the oscillator.
To observe the dynamic behaviour of forced van der pol Eq (5), use bifurcation plot and distinguish different modes like as periodic and quasiperiodic. These types of motions are

shown below.

Fig.9 shows the multiple or sub-multiple periodic system, which is known as quasiperiodic system, as compare this fig with fig.8; we observed that the both frequency and amplitude vary in time in quasiperiodic system.

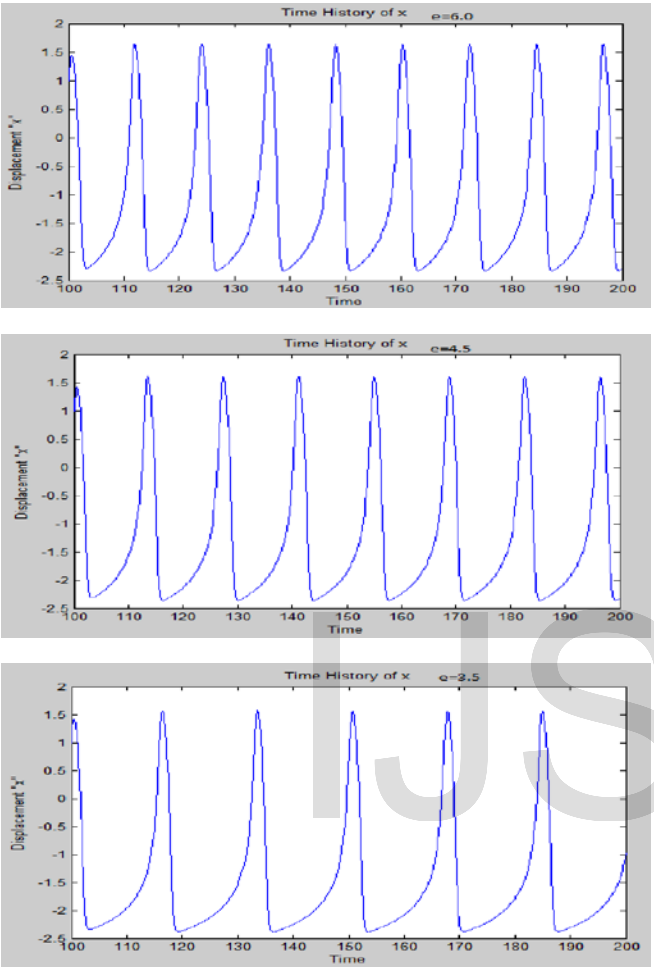

Fig. 5. Time series of Eq (3) for different values of e. e=6.0, 4.5 and 3.5 up to down respectively. The values of remaining parameter are α=1, d=3 and μ=1.

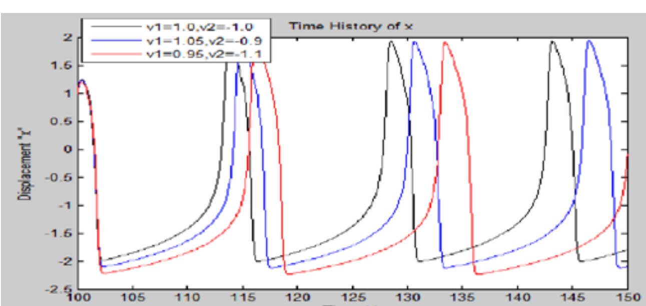

Fig. 6. Dependence of length of the diastolic period on the resting potential controlled by parameter v1 & v2. In all cases α=3, d=3, e=6

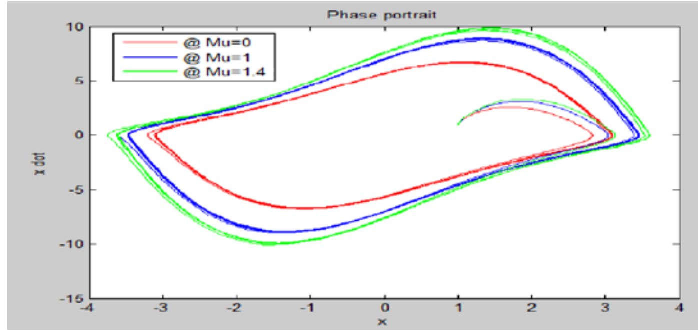

Fig. 7. Phase portrait of Eq (5) with different values of μ variable. The parameter α=1, F=10, Ω=1.1 and ω=1.

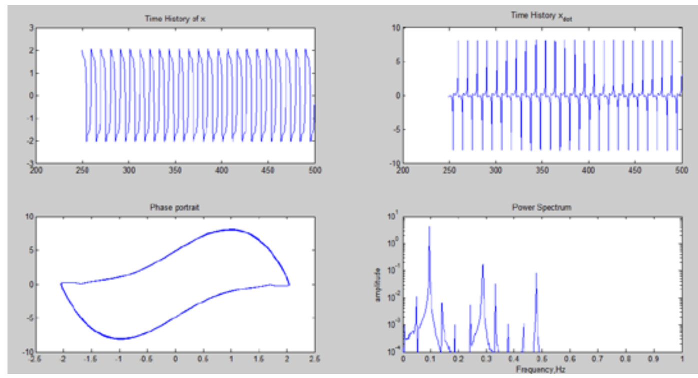

Fig. 8. Showing Time axis, derivative of time axis, phase portrait and frequency spectrum of periodic motion respectivily. Where α=5, μ=5, f=1, ω=1 and Ω=1.5

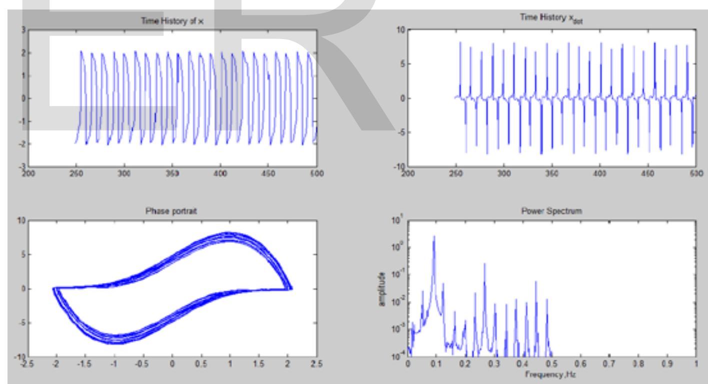

Fig. 9. Showing Time axis, derivative of time axis, phase portrait and frequency spectrum of quasi-periodic motion respectivily. Where α=5, μ=5, f=1, ω=1 and Ω=1.9

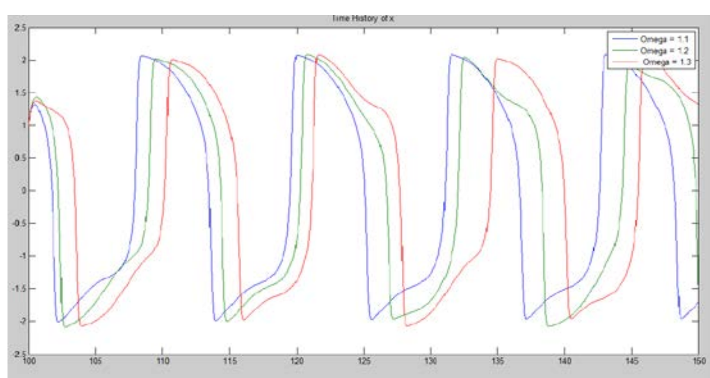

Fig. 10. Showing Time axis in different values of Ω at 1.1, 1.2 and 1.3 respectilvily. Whereas other parameter is α=5, μ=5, f=1 and ω=1

ISSN 2229-5518

5 CONCLUSION

In this paper we have reproduced several plot included forced vander pol oscillator. Such modification consisted in the introduction of limit cycle of the standard van der pol oscillation. Showing the unforced modified van der pol oscillation (mvdp) in time domain and forced modified vander pol oscillator both in time and frequency domain. The result reveal that as we change the frequency of the forced system, our system goes toward the qausi-periodic and then chaotic behaviors, whereas α&μ must be equal to obtain limit cycle from standard van der pol oscillator. For range of control parameter, α&μ must be same and not more than decade. Both ω and f should be equal to 1, whereas the Ω not more than 2.

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